MATH2230 Complex Variables with Application

Suggested Solution for HW5 & HW6

Sect. 33 No. 1

(a) $\mathrm{Log}(-ei) = \ln|-ei| + i\,\mathrm{Arg}(-ei) = 1 - \frac{\pi}{2}i$;

(b) $\mathrm{Log}(1-i) = \ln|1-i| + i\,\mathrm{Arg}(1-i) = \frac{1}{2}\ln 2 - \frac{\pi}{4}i$

Sect. 33 No. 2

(a) $\log e = \ln|e| + i\arg e = 1 + 2n\pi i \quad (n = 0, \pm1, \pm2, \cdots)$

(b) $\log i = \ln|i| + i\arg i = (\frac{1}{2}\pi + 2n\pi)i \quad (n = 0, \pm1, \pm2, \cdots)$

(c) $\log(-1+\sqrt{3}i) = \ln|-1+\sqrt{3}i| + i\arg(-1+\sqrt{3}i) = \ln 2 + i(\frac{2}{3}\pi + 2n\pi) \quad (n = 0, \pm1, \pm2, \cdots$

Sect. 36 No. 1

(a) $(1+i)^i = e^{i\log(1+i)} = e^{i[\ln\sqrt{2} + i(\frac{\pi}{4} + 2n\pi)]} = e^{-\frac{\pi}{4} + 2n\pi} e^{i\frac{\ln 2}{2}} \quad (n = 0, \pm1, \pm2, \cdots)$

(b) $\frac{1}{i^{2i}} = i^{-2i} = e^{-2i\log i} = e^{-2i[\ln|i| + i(\frac{\pi}{2} + 2n\pi)]} = e^{\pi + 4n\pi} \quad (n = 0, \pm1, \pm2, \cdots)$

Sect. 108 No. 8

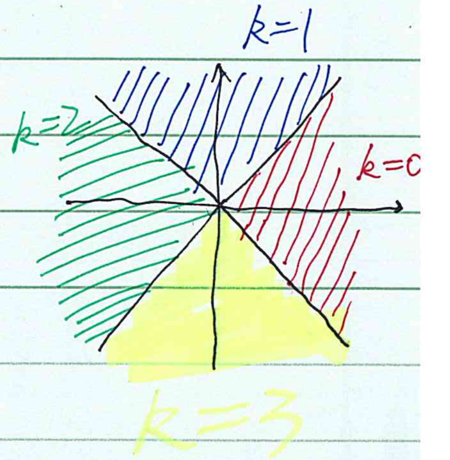
Solution. $F_k(z) = \sqrt[4]{r}\, e^{i\frac{\theta + 2k\pi}{4}} \quad (k = 0, 1, 2, 3) \quad \theta \in (-\pi, \pi]$

If $k=0$, then $F_0(z) = \sqrt[4]{r}\, e^{i\frac{\theta}{4}}$ $\qquad \frac{\theta}{4} \in (-\frac{\pi}{4}, \frac{\pi}{4}]$

If $k=1$, then $F_1(z) = \sqrt[4]{r}\, e^{i(\frac{\theta}{4} + \frac{\pi}{2})}$ $\qquad \frac{\theta}{4} + \frac{\pi}{2} \in (\frac{\pi}{4}, \frac{3}{4}\pi]$

If $k=2$, then $F_2(z) = \sqrt[4]{r}\, e^{i(\frac{\theta}{4} + \pi)}$ $\qquad \frac{\theta}{4} + \pi \in (\frac{3}{4}\pi, \frac{5}{4}\pi]$

If $k=3$, then $F_3(z) = \sqrt[4]{r}\, e^{i(\frac{\theta}{4} + \frac{3}{2}\pi)}$ $\qquad \frac{\theta}{4} + \frac{3}{2}\pi \in (\frac{5}{4}\pi, \frac{7}{4}\pi]$



The four branches are illustrated at right.

Thus, the four fourth roots of i under each branches are.

$e^{\frac{\pi}{8}i} \qquad e^{\frac{5}{8}\pi i} \qquad e^{\frac{9}{8}\pi i} \qquad e^{\frac{13}{8}\pi i}$